Title	A Simple Neural Network Model for Temporal Pulse Coding (Invariants of Dynamical Systems and Applications)
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Citation	数理解析研究所講究録 (1998), 1072: 162-173
Issue Date	1998-12
URL	http://hdl.handle.net/2433/62576
Right	
Type	Departmental Bulletin Paper
Textversion	publisher

数理解析研究所講究録
1072巻 1998年 162-173

A Simple Neural Network Model for Temporal Pulse Coding

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1 Introduction

In recent years there has been renewal of interest in the basic coding principle of the brain, how information is transmitted among neurons. Today, there are two major hypothesis: rate coding, where the mean firing rate is the carrier of information, and temporal coding, where timing of individual pulses are the carriers of information. Rate coding hypothesis has been the established theory for a long time and most model approaches as well as many experiments on biological networks have been based on this idea. This hypothesis was widely accepted due to the existence of a quantitative relationship between the firing rates of single cortical neurons and psychophysical judgments made by behaving monkeys(Barlow 1987; Newsome 1989; Britten 1992).

On the other hand, spatio-temporal coding has been recently brought to light by both biologists and theorists. Softky and Koch (1993) reported that the inter-spike intervals of cortical pyramidal cells are highly variable and concluded that neurons function as coincidence detectors which indicates temporal coding. Furthermore, Vaadia *et.al* observed the dynamical modulation of temporal relations between the discharges of neurons in the monkey prefrontal cortex in response to contexts or external events, leading to the idea that information is contained in the spike timing(Vaadia 1995). On the theoretical side, Abeles proposed the notion of "synfire chains" which assures the timing accuracy of neuronal firings observed in brains of behaving monkeys with rather unreliable neurons(Abeles 1993b). Besides this, many other experimental results and theoretical considerations support temporal pulse coding in the brain(Sakurai 1993; Tsukada *et al.* 1992).

We believe that the truth lies somewhere in the between of pure rate coding and pure temporal coding. However, we feel that it is also necessary to simplify things in order to carry out theoretical studies, such as viewing the coding of the brain from the two extremes, the rate coding and temporal coding hypothesis. The problem is that there seems to be only few fundamental models which give a mathematical description of the underlying dynamics in temporal pulse coding. Whereas in the rate coding paradigm, say, we have the Hopfield model(Hopfield 1982). Hopfield introduced the notion of "energy" in a single layer neural network and showed that the neuronal dynamics evolves so that energy decreases with time.

The purpose of this paper is to introduce a simple model which describes mathematically the ongoing dynamics of temporal pulse coding. That is to say, we will try to purely extract the effects of incident pulse timing on network dynamics and also make it simple enough for theoretical analysis. To be more specific, we eliminate the effects of spatial summation of neuronal activities, which was the key to rate coding, by using "uniform" synaptic strength on a mutually connected network. Also we make several assumptions so that we are able to define "network state"in such continuous time systems and thus describe the dynamics by iteration of maps.

We will first give the descriptions of the proposed model and next analyze the model by means of numerical simulation from the aspects of dynamical stability and basins of attraction. Furthermore, we introduce a learning rule which increases the stability and basin of attraction of attractors in our proposed model. Final section is assigned to discussions.

2 Neural Network Model

The network model is a deterministic point process model based on temporal pulse coding. That is to say, pulses actually propagate among neurons with finite delay, and neurons work as coincidence detectors of incident pulses. Furthermore, in order

to extract the effect of incident pulse timings on neuronal interaction, we assume that neurons are mutually connected with "uniform" synaptic efficacy. This corresponds to the case where only one pattern is embedded in the network by the Hebbian learning rule, which is not very interesting from the view point of classical rate coding. However, in a temporal pulse coding network model, by assuming random time delay for pulse propagation, it has been reported to show various dynamics(Watanabe and Aihara (1997), Watanabe *et. al* (1998)).

Before giving equations and going into the details of the model, we take up the two assumptions which we make to construct a "simple as possible" model. The first is that neuronal firings are localized in time. We realize this by setting the variance of pulse propagation delay small enough so that pulses sent from a group of firing neurons do not spread out in time and overlap with pulses sent out from other groups of neuronal firings. We call this group of pulses, "pulse group". As a consequence, localized neuronal firings become a function of the previous localized neuronal firings, that is to say, it becomes possible to describe the network dynamics in discrete steps of pulse groups.

The second assumption is to fix the number of firing neurons resulting from a single pulse group. We assume some kind of a negative feedback loop which keeps a moderate rate of activity in the network. As a consequence, the network state becomes a fixed dimensional vector and the dynamics can be written as iterations of maps.

Now that we are ready, we will go onto the details of the model, first in the general case. We define $\mathbf{t}^f(k)$ as the neuronal firing time vector with firing time of N neurons in the kth pulse group as its components. As we stated earlier, only n_f neurons fire in a single pulse group, so n_f components have actual firing time as its value and rest will be "-1" indicating that the neuron did not fire in the k th pulse group. From this neuronal firing vector, matrix of pulse arrival time in the $k+1$th pulse group $\mathbf{T}^p(k+1)$ is given,

$$\mathbf{T}^p(k+1) = F(\mathbf{t}^f(k), \mathbf{D}), \tag{1}$$

where function F adds pulse propagation delay given by the delay matrix $\mathbf{D}$ to the neuronal firing time as follows,

$$t_{ij}^p(k+1) = \begin{cases} t_j^f(k) + d_{ij}, & \text{if} t_j^f \geq 0, \\ -1, & \text{if} t_j^f = -1. \end{cases} \tag{2}$$

Here, $t_{ij}^p(k+1)$ and d_{ij} are the components of matrix $\mathbf{T}^p(k+1)$ and $\mathbf{D}$, denoting pulse arrival time and pulse propagation delay from pre-neuron j to post-neuron i, respectively.

Given the pulse arrival time of all neurons in the network, the neuronal firing time of the $k+1$th pulse group is determined,

$$\mathbf{t}^f(k+1) = G(\mathbf{T}^p(k+1)). \tag{3}$$

Here, the role of function G is to determine "which neurons" and "when" to fire given the distribution of pulses. Thus the selection of this function decides the property of a single neuron and the negative feedback loop for stabilizing activity. Taking equations (1) and (3) into account, the overall dynamics of our model can be written as

$$\mathbf{t}^f(k+1) = G \circ F(\mathbf{t}^f(k), \mathbf{D}). \tag{4}$$

Next we describe the definite case used in the following simulations where, as the most simple case, we choose the number of firing neurons in a pulse group $n_f = 2$ and function G as described below.

The first part of function G is to decide which neurons to fire. Defining $\phi(1,k)$ and $\phi(2,k)$ as the index of two firing neurons in the kth pulse group ($\phi(1,k) < \phi(2,k)$), we

choose the next two firing neurons with the smallest two incident pulse interval(IPI) as,

$$IPI_i(k+1) = |t^p_{\phi(1,k),i}(k+1) - t^p_{\phi(2,k),i}(k+1)| = |\Delta t^f(k) + d_{\phi(1,k),i} - d_{\phi(2,k),i}|, \quad (5)$$

$$IPI_{\phi(1,k+1)}(k+1), IPI_{\phi(2,k+1)}(k+1) < IPI_i(k+1) \ \ (i = 1, ..., N, i \neq \phi(1,k+1), \phi(2,k+1)), \quad (6)$$

where $\Delta t^f(k) = t^f_{\phi(1,k)}(k) - t^f_{\phi(2,k)}(k)$. This method of choosing neurons to fire reflects the idea that neurons act as coincidence detectors of incident pulses in temporal pulse coding.

Now that we know which neurons to fire in the $k+1$ th pulse group, the next role of function G is to determine "when" to fire. The firing time is a function of the local information available to a neuron, the pulse arrival time, as follows,

$$t^f_i(k+1) = (t^p_{\phi(1,k),i} + t^p_{\phi(2,k),i})/2 + a|t^p_{\phi(1,k),i} - t^p_{\phi(2,k),i}|^\gamma \ \ (i = \phi(1,k+1), \phi(2,k+1)), \quad (7)$$

where the first term of the right hand side takes the average of arrival time of the two pulses and the second term describes the firing delay depending on IPI. Larger the IPI, larger the firing delay. The two parameters a and γ characterizes the property of the firing delay. These two values and the delay matrix are the only parameters in our model and as we will see in the following section, especially a has a large influence on the network dynamics.

From equations(2) and (7), we get the following equation which corresponds to eq. (4) in the general case,

$$t^f_i(k+1) = (t^f_{\phi(1,k)} + t^f_{\phi(2,k)})/2 + (d_{\phi(1,k),i} + d_{\phi(2,k),i})/2$$

$$+ a|t^f_{\phi(1,k)} - t^f_{\phi(2,k)} + d_{\phi(1,k),i} - d_{\phi(2,k),i}|^\gamma$$
$$(i = \phi(1,k+1), \phi(2,k+1)). \tag{8}$$

Moreover, subtracting $t^f_{\phi(1,k+1)}(k+1)$ from $t^f_{\phi(2,k+1)}(k+1)$, we get

$$\begin{aligned}\Delta t^f(k+1) &= (d_{\phi(1,k),\phi(1,k+1)} + d_{\phi(2,k),\phi(1,k+1)} - d_{\phi(1,k),\phi(2,k+1)} - d_{\phi(2,k),\phi(2,k+1)})/2 \\ &+ a|\Delta t^f(k) + d_{\phi(1,k),\phi(1,k+1)} - d_{\phi(2,k),\phi(1,k+1)}|^\gamma \\ &- a|\Delta t^f(k) + d_{\phi(1,k),\phi(2,k+1)} - d_{\phi(2,k),\phi(2,k+1)}|^\gamma. \end{aligned} \tag{9}$$

Notice that from equations (6) and (9), the iteration of the map only depends on $\Delta t^f(k)$ and $\phi(1,k)$,$\phi(2,k)$. Therefore we can consider the network state of the k th pulse group as the following three dimensional vector,

$$\mathbf{s}(k) = \{\Delta t^f(k), \phi(1,k), \phi(2,k)\}. \tag{10}$$

Furthermore, without losing generality, we can map this three dimensional state space onto one dimension which makes it possible to view the mapping,

$$S(k) = \Delta t^f(k) + \psi\{\phi(1,k), \phi(2,k)\}, \tag{11}$$

where ψ is a function such as,

$$\psi(i,j) = 2\Delta t^f_{max}\{(2N-1)(i-1)/2 + j - i\}. \tag{12}$$

Taking equations (9)(11) into account, the overall dynamics of our network with $n_f = 2$ becomes a one dimensional return map.

$$S(k+1) = \Omega(S(k), \mathbf{D}), \tag{13}$$

$$S(k+1) = (d_{\phi(1,k),H_1(S(k))} + d_{\phi(2,k),H_1(S(k))} - d_{\phi(1,k),H_2(S(k))} - d_{\phi(2,k),H_2(S(k))})/2$$

$$+ a|S(k) - \psi\{\phi(1,k), \phi(2,k)\} + d_{\phi(1,k),H_1(S(k))} - d_{\phi(2,k),H_1(S(k))}|^\gamma$$
$$- a|S(k) - \psi\{\phi(1,k), \phi(2,k)\} + d_{\phi(1,k),H_2(S(k))} - d_{\phi(2,k),H_2(S(k))}|^\gamma$$
$$+ \psi\{H_1(S(k)), H_2(S(k))\}. \qquad (14)$$

where $H_i(S(k))$ $(i = 1, 2)$ is a function which determines what neurons to fire from the previous network state using equation (6).

3 Simulation results

3.1 Change of parameter "γ"- piecewise linear and nonlinear

A network of $N = 4$ neurons with uniform random delay $(8 < d_{ij} < 12)$ is studied in this section to investigate the change of dynamics with parameter "γ".

First, we will look at the return map of network states when γ is 2. In this case, eq.(14) becomes,

$$\begin{aligned} S(k+1) &= 2a(d_{\phi(1,k),H_1(S(k))} - d_{\phi(2,k),H_1(S(k)))} - d_{\phi(1,k),H_1(S(k))} + d_{\phi(2,k),H_1(S(k))})S(k) \\ &+ (d_{\phi(1,k),H_1(S(k))} + d_{\phi(2,k),H_1(S(k))} - d_{\phi(1,k),H_2(S(k))} - d_{\phi(2,k),H_2(S(k))})/2 \\ &+ (d_{\phi(1,k),H_1(S(k))} - d_{\phi(2,k),H_1(S(k)))})^2 - (d_{\phi(1,k),H_2(S(k))} - d_{\phi(2,k),H_2(S(k)))})^2 \\ &+ \psi\{H_1(S(k)), H_2(S(k))\} = a\alpha(S(k), \mathbf{D})S(k) + \beta(S(k), \mathbf{D}), \qquad (15) \end{aligned}$$

and since $H_i(S(k)$ stays constant for small changes of $S(k)$, it is a piecewise linear return map. Figure(1)(a) shows the actual return map for $\gamma = 2$.

On the other hand, Figure(1)(b) shows the case for $\gamma = 1.2$ where it becomes a piecewise nonlinear map.

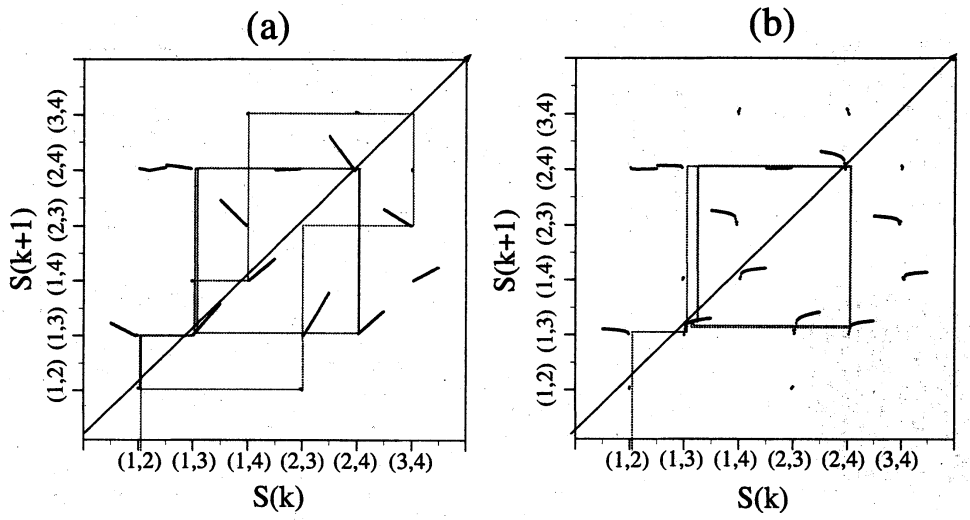

Figure 1: Return plot for (a) $\gamma = 2.0$, piece-wise linear case and (b) $\gamma = 1.2$, piece-wise nonlinear case($N = 4$, $n_f = 2$, $\bar{d} = 10.0 < d >= 2.0$)

3.2 Change of parameter "a" - periodic, chaotic and complex dynamics

In this subsection, we investigate the change of network dynamics with parameter a which first appears in eq.(7). As we can see from eq.(14), parameter a gives the mean inclination of the map. Therefore, increasing a works to raise the average gradient of the map, hence, the dynamics becomes less stable.

The neuronal firing of the network with two different a is given in figure(2). As we can see, the network dynamics is periodic for $a = 0.5$ and chaotic for $a = 5.0$. Here we choosed 1.2 for γ since the piecewise linear case ($\gamma = 2.0$) tends to break the assumption which says that pulse groups should not overlap when a takes large values.

To get a whole grasp of the change of network behavior with a, we give a diagram of bifurcation and lyapunov exponents in figure(3). Here, the lyapunov exponent is defined as

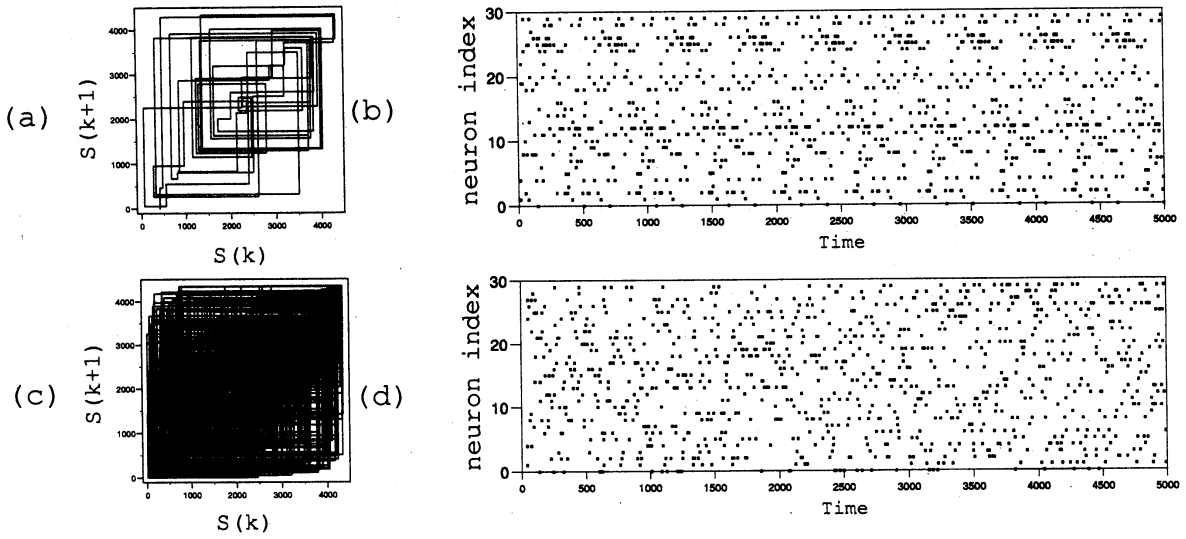

Figure 2: Return plot and Raster diagram (a),(b) periodic dynamics : a=0.05 (c),(d) chaotic dynamics: a=5.0 ($N = 30$, $n_f = 2$, $\bar{d} = 10.0 < d >= 2.0$ $\gamma = 1.2$)

$$\lambda = 1/K \sum_{k=0}^{K-1} log|dS(k+1)/dS(k)|, \tag{16}$$

$$\begin{aligned} dS(k+1)/dS(k) &= a\gamma\{|S(k) - \psi\{\phi(1,k),\phi(2,k)\} + d_{\phi(1,k),H_1(S(k))} - d_{\phi(2,k),H_1(S(k))}|^{\gamma-1} \\ &\quad - |S(k) - \psi\{\phi(1,k),\phi(2,k)\} + d_{\phi(1,k),H_2(S(k))} - d_{\phi(2,k),H_2(S(k))}|^{\gamma-1}\} \\ &= a\gamma\{|t^p_{\phi(2,k),\phi(1,k+1)} - t^p_{\phi(1,k),\phi(1,k+1)}|^{\gamma-1} - |t^p_{\phi(1,k),\phi(2,k+1)} - t^p_{\phi(2,k),\phi(2,k+1)}|^{\gamma-1} \end{aligned} \tag{17}$$

Calculation for a single a was started from 100 distinct initial states and plotted after transient($k > 10000$). As parameter a increases, the lyapunov exponent gradually rises and from the bifurcation diagram we can see that the network dynamics becomes chaotic. We should also notice the gap between multiple lyapunov exponents for single a where they are negative. This indicates the existence of multiple stable

attractors. Furthermore, when a becomes larger($a > 2.5$), periodic attractors and chaotic attractors co-exist.

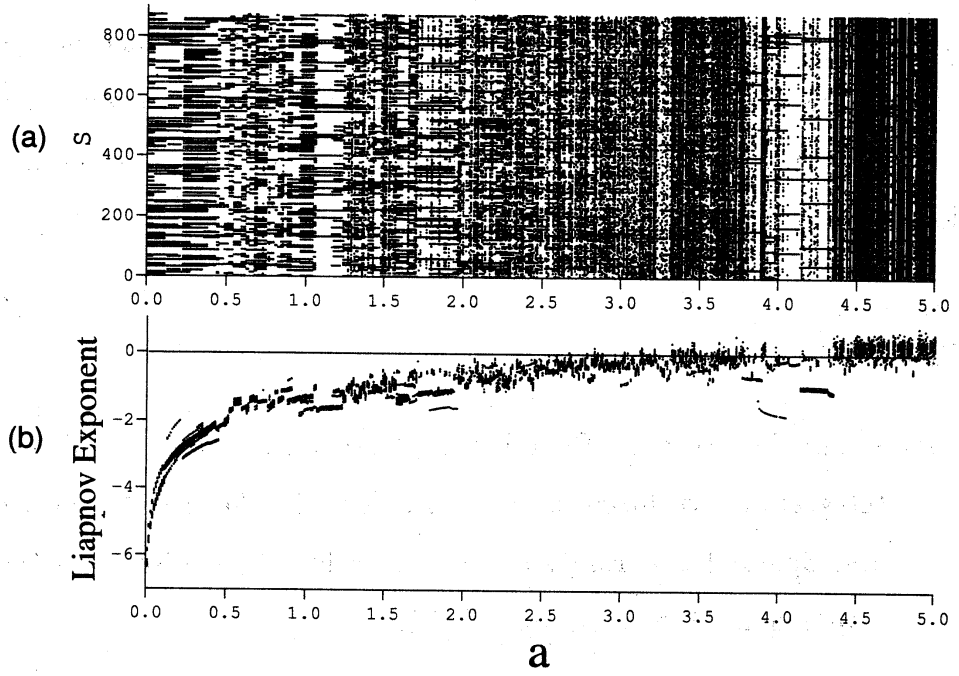

Figure 3: (a) Bifurcation diagram of neuronal parameter a. (b) plot of liapnov exponent ($N = 30$, $n_f = 2$, $\bar{d} = 10.0 < d >= 2.0$ $\gamma = 1.2$)

4 Discussion

We have introduced a simple pulse-coded model. The dynamics of the model was clarified using a return map and the lyapunov exponent, where existence of periodic and chaotic attractors were confirmed. It is also important to note that multiple stable attractors exist in our model. We may consider these attractors as "memories", as in the Hopfield model. We have also proposed a learning rule which decreases the lyapunov exponent of an attractor and hence increases the stability and the basin of attraction.

As we have stated earlier, this paper concentrated on the analysis of pure temporal coding, in other words, we investigated only the effect of pulse timing on neuronal interactions. Neuronal interactions, say, how neurons decide when to fire given the outputs of other neurons in the network, is also a function of what neurons fired. This is realized in the distribution of synaptic weights in usual models, whereas in our model we used an uniform distribution. Combining temporal and spatial effect on neuronal interaction will be the subject of our future study.

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